

Linear and Nonlinear Simulation for Quadrotor Dynamics Encompassing Induced Disturbance

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Abstract

Quadrotor is a rotorcraft that has four rotors, usually symmetrical and arranged in some specific configuration, i.e. 'plus' and 'cross' configuration. To test a new control system for quadrotor, it is necessary to put it on board first and have a flight test. However, controlling a quadrotor is a hard task, hence the risk of crash is high. This risk can be reduced by creating a simulation model that represents the dynamics of quadrotor accurately. This paper aims to provide the simulation model of a quadrotor. To increase accuracy of this model, the disturbance induced by the motion of the quadrotor were also taken into account. The simulation was built in Simulink based on the kinematics and dynamics equation of quadrotor both in linear and nonlinear simulation. In most cases, the attitude response (roll, pitch, and yaw) for both simulations were nearly the same. The most notable difference was for the elevation response in which the linear simulation tended to be unable to capture the deviation, while the nonlinear simulation was able to.

Keywords : *Quadrotor, Linear simulation, Nonlinear simulation, Disturbance*

Abstrak

Quadrotor adalah sebuah pesawat nirawak yang mempunyai empat buah rotor, biasanya simetris dan disusun dalam beberapa konfigurasi tertentu, yaitu konfigurasi 'plus' dan 'silang'. Untuk menguji sistem kendali baru pada quadrotor, sistem tersebut perlu dipasang terlebih dahulu dan dilakukan uji terbang. Namun, mengendalikan quadrotor adalah hal yang sulit, sehingga risiko kecelakaannya tinggi. Risiko ini dapat dikurangi dengan membuat model simulasi yang

merepresentasikan dinamika quadrotor secara akurat. Tulisan ini bertujuan untuk memberikan model simulasi quadrotor. Untuk meningkatkan akurasi model ini, gangguan yang disebabkan oleh gerakan quadrotor juga diperhitungkan. Simulasi dibangun di Simulink berdasarkan persamaan kinematika dan dinamika quadrotor baik dalam simulasi linier maupun nonlinier. Dalam kebanyakan kasus, respon sikap (roll, pitch, dan yaw) untuk kedua simulasi hampir sama. Perbedaan yang paling menonjol adalah pada respon elevasi dimana simulasi linier cenderung tidak mampu menangkap penyimpangan, sedangkan simulasi nonlinier mampu

Kata Kunci: Quadrotor, Simulasi linier, Simulasi nonlinier, Gangguan

INTRODUCTION

In the last century, the development of technology in the world has been increasing rapidly and widely, compared to centuries ago, thanks to the industrial revolution era. Humans are competing to invent new tools or methods either for economic reasons or just to fulfill their curiosity. The aerospace industry is not an exception; It grew initially due to the World War era to provide fighter aircraft, followed by the Cold War between USA and USSR that led to major development of spacecraft technology.

Nowadays, the range of products aircraft industries provide is becoming more extensive. Particularly in recent decades, the development of Unmanned Aerial Vehicle (UAV) has been drawing the attention of many parties in the world. Since it is pilotless, its utilization is flexible, let alone the fact that it is considerably small and light (Al-Shareeda et al., 2023). It was initially developed for military purposes (Ibrahim Jenie et al., 2018), but its function is

expanding to non-military purposes as well (Gonzalez-Dugo et al., 2013), such as rescue missions, surveillance, mapping, photography, etc. (Rahani & Priyambodo, 2019; Suthanthira Vanitha et al., 2020). It is also giving a major impact in economy, creating more than 70,000 job opportunities in USA (Jenkins & Vasigh, 2013).

One of the most popular type of UAVs is rotorcraft. A rotorcraft relies on their rotors that provide upward thrust to compensate the weight. Rotorcraft provides the ability to hover and vertical take-off and landing that are suitable for some specific mission that requires the aircraft to stay at one point for some time, or when the runway available is limited (Idrissi et al., 2022). A rotorcraft is also able to fly backwards and sideways that makes it more flexible when maneuvering in the air (Abdelmaksoud et al., 2020).

A more specific type of rotorcraft is a quadrotor; It is a rotorcraft that has four rotors, usually symmetrical and arranged in some specific configuration, i.e. ‘plus’ and ‘cross’ configuration. The latter configuration is currently more preferred as it gives higher momentum, hence making the maneuverability of the quadrotor increases, since all the rotors are involved to achieve the desired maneuver (Partovi et al., 2012). However, regardless of the configuration, quadrotor is intrinsically unstable (Lee et al., 2023). Its dynamics is too fast for humans; hence it is difficult for humans to fly it manually. The control of quadrotor is an issue both practically and theoretically. Therefore, a program to automatically control this aerial vehicle is a must (Sadr et al., 2014).

To test a new control system for quadrotor, it is necessary to put it on board first and have a flight test. However, since it is harder to control quadrotor, the risk of crash is higher. Fortunately, this risk can be reduced by creating a simulation model that represents the dynamics of quadrotor accurately (Omar, 2022). The simulation model can be obtained by evaluating the kinematics and dynamics equation of a quadrotor. By having this simulation model, the new controller proposed can be tested first to the simulation, and refined there, before implement it to the real targeted quadrotor.

This paper aims to provide the simulation model of a quadrotor. To increase accuracy of this model, the disturbance induced by the motion of the quadrotor were also taken into account. The simulation was built in Simulink based on the kinematics and dynamics equation of quadrotor. Both linear and nonlinear approach simulation were developed and the results were compared.

RESEARCH METHOD

This research was conducted numerically in Simulink Matlab. Firstly, the kinematics and dynamics model of a quadrotor was developed from its fundamental equations. Based on those models, simulations were built in Simulink, either in linear approach or nonlinear approach.

Reference Frame and Rotation Matrix

Let us define two fundamental reference frames used for this model (see **equa 1**); They are Earth reference frame (indicated by orange-colored reference frame) and Body reference frame (indicated by red-colored reference frame). The angles corresponding to a rotation around each axis (x, y, and z) are roll (ϕ), pitch (θ), and yaw (ψ) angle. The earth reference frame and body reference frame would be related by the rotation matrix $R_{zyx}(\phi, \theta, \psi)$ as represented in **Equation (1)**.

$$R_{zyx} = \quad (1)$$

$$\begin{bmatrix} c(\theta)c(\psi) & s(\phi)s(\theta)c(\psi) - c(\phi)c(\psi) & c(\phi)s(\theta)c(\psi) + s(\phi)s(\psi) \\ c(\theta)s(\psi) & s(\phi)s(\theta)s(\psi) + c(\phi)c(\psi) & c(\phi)s(\theta)s(\psi) - s(\phi)c(\psi) \\ -s(\theta) & s(\phi)c(\theta) & c(\phi)c(\theta) \end{bmatrix}$$

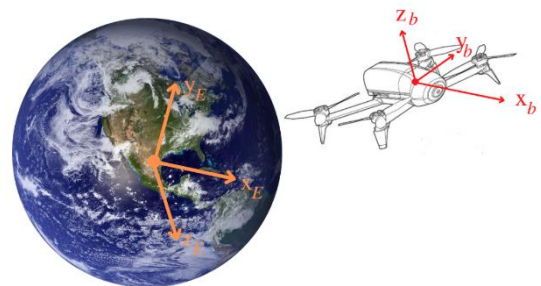


Figure 1 Reference Frames

Kinematics Model

Consider $[x \ y \ z \ \phi \ \theta \ \psi]^T$, a vector containing the linear and angular position of the quadrotor in the earth frame, and $[u \ v \ w \ p \ q \ r]^T$, a vector containing the linear and angular velocities in the body frame. The dynamics of the quadrotor in those two

reference frames are related by these equations (FRANCESCO SABATINO, 2015):

$$[\dot{x} \ \dot{y} \ \dot{z}]^T = \mathbf{R}_{zyx} \cdot [u \ v \ w]^T \quad (2)$$

$$[\dot{\phi} \ \dot{\theta} \ \dot{\psi}]^T = \mathbf{T} \cdot [p \ q \ r]^T \quad (3)$$

Where (Basri et al., 2015)

$$\mathbf{T} = \begin{bmatrix} 1 & s(\phi)t(\theta) & c(\phi)t(\theta) \\ 0 & c(\phi) & -s(\phi) \\ 0 & \frac{s(\phi)}{c(\theta)} & \frac{c(\phi)}{c(\theta)} \end{bmatrix} \quad (4)$$

Therefore, the full model of the quadrotor kinematics is:

$$\begin{aligned} \dot{x} = & u[c(\theta)c(\psi)] \\ & + v[s(\phi)s(\theta)c(\psi) \\ & - c(\phi)c(\psi)] \\ & + w[c(\phi)s(\theta)c(\psi) \\ & + s(\phi)s(\psi)] \end{aligned} \quad (5)$$

$$\begin{aligned} \dot{y} = & u[c(\theta)s(\psi)] \\ & + v[s(\phi)s(\theta)s(\psi) \\ & + c(\phi)c(\psi)] \\ & + w[c(\phi)s(\theta)s(\psi) \\ & - s(\phi)c(\psi)] \end{aligned} \quad (6)$$

$$\dot{z} = u[-s(\theta)] + v[s(\phi)c(\theta)] + w[c(\phi)c(\theta)] \quad (7)$$

$$\dot{\phi} = p + q[s(\phi)t(\theta)] + r[c(\phi)t(\theta)] \quad (8)$$

$$\dot{\theta} = q[c(\phi)] - r[s(\phi)] \quad (9)$$

$$\dot{\psi} = q \frac{s(\phi)}{c(\theta)} + r \frac{c(\phi)}{c(\theta)} \quad (10)$$

Dynamics Model

Based on Newton's Law, the relations of total force and torque acting on quadrotor are:

$$m \left(\begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} \times \begin{bmatrix} u \\ v \\ w \end{bmatrix} + \begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{bmatrix} \right) = \mathbf{F}_B \quad (11)$$

$$\mathbf{I} \cdot \begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} + \mathbf{I} \times \left(\mathbf{I} \cdot \begin{bmatrix} p \\ q \\ r \end{bmatrix} \right) = \mathbf{M}_B \quad (12)$$

Where $\mathbf{F}_B = [F_x \ F_y \ F_z]^T$ is the total force vector acting on body reference frame, $\mathbf{M}_B = [M_x \ M_y \ M_z]^T$ is the total moment vector acting

on body reference frame, and \mathbf{I} is moment of inertia diagonal matrix (assuming the quadrotor is symmetrical):

$$\mathbf{I} = \begin{bmatrix} I_x & 0 & 0 \\ 0 & I_y & 0 \\ 0 & 0 & I_z \end{bmatrix} \quad (13)$$

Hence, the dynamic of the quadrotor is expressed as follows:

$$F_x = m(\dot{u} + qw - rv) \quad (14)$$

$$F_y = m(\dot{v} - pw + ru) \quad (15)$$

$$F_z = m(\dot{w} + pv - qu) \quad (16)$$

$$M_x = \dot{p}I_x - qrI_y + qrI_z \quad (17)$$

$$M_y = \dot{q}I_y + prI_x - prI_z \quad (18)$$

$$M_z = \dot{r}I_z - pqI_x + pqI_y \quad (19)$$

On the other hand, the total force and torque can be expanded more as follows:

$$F_x = W[s(\theta)] + f_{wx} \quad (20)$$

$$F_y = -W[c(\theta)s(\phi)] + f_{wy} \quad (21)$$

$$F_z = -W[c(\theta)c(\phi)] + f_{wz} + f_t \quad (22)$$

$$M_x = \tau_x + \tau_{wx} \quad (23)$$

$$M_y = \tau_y + \tau_{wy} \quad (24)$$

$$M_z = \tau_z + \tau_{wz} \quad (25)$$

Where $W = mg$ is the weight of the quadrotor; f_{wx}, f_{wy}, f_{wz} are the disturbance forces induced by the motion of the quadrotor; f_t is the total thrust generated by the rotors; τ_x, τ_y, τ_z are the torque generated by the rotors; and $\tau_{wx}, \tau_{wy}, \tau_{wz}$ are the disturbance torques induced by the motion of the quadrotor.

The disturbance forces and torques induced by the motion of the quadrotor are defined as follows (Ibrahim Jenie et al., 2018):

$$f_{wz} = \sum f_w(w + px_i - qy_i) = 4f_w w \quad (26)$$

$$\tau_{wx} = \sum f_w y_i (w + py_i - qx_i) = 4f_w l_y^2 p \quad (27)$$

$$\tau_{wy} = - \sum f_w x_i (w + py_i - qx_i) = 4f_w l_x^2 q \quad (28)$$

$$\tau_{wz} = \sum \tau_{zw}(w + py_i - qx_i) = 0 \quad (29)$$

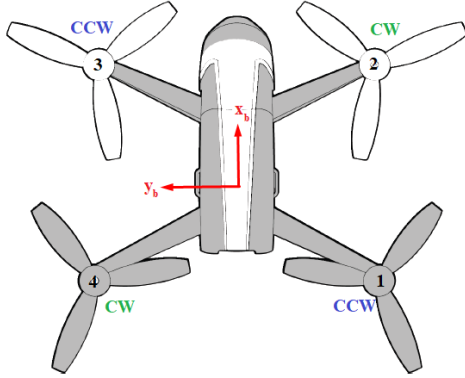


Figure 2 Quadrotor configuration

Where i index represents the corresponding index of rotor (see **Figure 2** for quadrotor configuration); lx_i and ly_i represent the position of corresponding rotor; and f_w is Thrust coefficient with respect to w . The disturbances emerge mainly due to the effective upward velocity w that is lower because of the motion of the quadrotor. **Figure 3** below depicts the schematic of how the disturbances are induced.

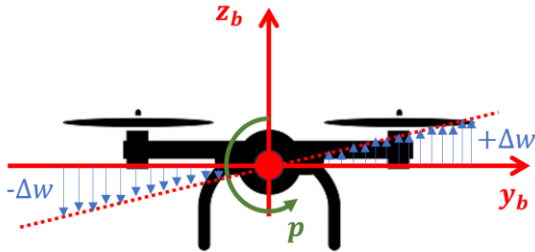


Figure 3 The schematic of induced disturbance

Consider the quadrotor rotating around x axis. Due to this rotation, the rotors, which are located at either further side of y axis, will experience either a rise or a drop of upward velocity w . Hence, the change of w due to roll rate q is:

$$\Delta w_p^x = py_i \quad (30)$$

By using the same logic, the change of w as a result of pitch rate q is:

$$\Delta w_p^x = -qx_i \quad (31)$$

Therefore, the effective w is:

$$w_{eff}^x = w + py_i - qx_i \quad (32)$$

The expression above is the effective w that leads to disturbance of torque around x axis τ_{wx} . By doing the same, the disturbance forces and torques induced by the motion of the quadrotor are obtained as presented previously.

Rotor Dynamics

Consider the configuration of the rotors as depicted in **Figure 2**. The dynamics of the rotors is expressed as:

$$\tau_x = f_{t\delta} l_y (-\delta_1 - \delta_2 + \delta_3 + \delta_4) \quad (33)$$

$$\tau_y = f_{t\delta} l_x (\delta_1 - \delta_2 - \delta_3 + \delta_4) \quad (34)$$

$$\tau_z = \tau_{z\delta} (-\delta_1 + \delta_2 - \delta_3 + \delta_4) \quad (35)$$

$$f_t = f_{t\delta} (\delta_1 + \delta_2 + \delta_3 + \delta_4) \quad (36)$$

Where δ is the throttle magnitude; $f_{t\delta}$ is Thrust coefficient with respect to throttle; $\tau_{z\delta}$ is torque coefficient with respect to throttle; l_x is the arm length of quadrotor in x axis; and l_y is the arm length quadrotor in y axis.

By assuming that attitude disturbances are low, we can take assumption that $c(\beta) = 1$ and $s(\beta) = 0$. Rearranging and substituting the kinematics and dynamics equations of quadrotor, we obtained:

$$\dot{\phi} \approx p + r\theta + q\phi\theta \quad (37)$$

$$\dot{\theta} \approx q - r\phi \quad (38)$$

$$\dot{\psi} \approx r + q\phi \quad (39)$$

$$\dot{p} \approx \frac{I_y - I_z}{I_x} r q + \frac{\tau_x + \tau_{wx}}{I_x} \quad (40)$$

$$\dot{q} \approx \frac{I_z - I_x}{I_y} p r + \frac{\tau_y + \tau_{wy}}{I_y} \quad (41)$$

$$\dot{r} \approx \frac{I_x - I_y}{I_z} p q + \frac{\tau_z + \tau_{wz}}{I_z} \quad (42)$$

$$\dot{u} \approx rv - qw + g\theta + \frac{f_{wx}}{m} \quad (43)$$

$$\dot{v} \approx pw - ru - g\theta + \frac{f_{wy}}{m} \quad (44)$$

$$\dot{w} \approx qu - pv + g + \frac{f_{wz} + f_t}{m} \quad (45)$$

$$\dot{x} \approx w(\phi\psi + \theta) - v(\psi - \phi\theta) + u \quad (46)$$

$$\dot{y} \approx v(1 + \phi\psi\theta) - w(\phi - \psi\theta) + u\psi \quad (47)$$

$$\dot{z} \approx w - u\theta + v\phi \quad (48)$$

The set of equations above will be used for **nonlinear model simulation**. However, we can simplify further those equations by neglecting higher order terms. Hence the equations will be:

$$\dot{\phi} \approx p \quad (49)$$

$$\dot{\theta} \approx q \quad (50)$$

$$\dot{\psi} \approx r \quad (51)$$

$$\dot{p} \approx \frac{\tau_x + \tau_{wx}}{I_x} \quad (52)$$

$$\dot{q} \approx \frac{\tau_y + \tau_{wy}}{I_y} \quad (53)$$

$$\dot{r} \approx \frac{\tau_z + \tau_{wz}}{I_z} \quad (54)$$

$$\dot{u} \approx g\theta + \frac{f_{wx}}{m} \quad (55)$$

$$\dot{v} \approx g\theta + \frac{f_{wy}}{m} \quad (56)$$

$$\dot{w} \approx \frac{f_{wz} + f_t}{m} \quad (57)$$

$$\dot{x} \approx u \quad (58)$$

$$\dot{y} \approx v \quad (59)$$

$$\dot{z} \approx w \quad (60)$$

Moreover, those equations can be arranged in state space form as presented as follows. This state-space form of quadrotor dynamics will be used for **Linear model simulation**

$$\begin{pmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \\ \dot{p} \\ \dot{q} \\ \dot{r} \\ \dot{u} \\ \dot{v} \\ \dot{w} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{f_{wx}}{I_x} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{f_{wy}}{I_y} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & g & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -g & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4f_w \end{pmatrix} \begin{pmatrix} \phi \\ \theta \\ \psi \\ p \\ q \\ r \\ u \\ v \\ w \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -\frac{f_{\delta 1} l_y}{I_x} & -\frac{f_{\delta 1} l_y}{I_x} & \frac{f_{\delta 1} l_y}{I_x} & \frac{f_{\delta 1} l_y}{I_x} \\ \frac{f_{\delta 2} l_y}{I_x} & -\frac{f_{\delta 2} l_y}{I_x} & -\frac{f_{\delta 2} l_y}{I_x} & \frac{f_{\delta 2} l_y}{I_x} \\ -\frac{\tau_{z\delta}}{I_z} & \frac{\tau_{z\delta}}{I_z} & -\frac{\tau_{z\delta}}{I_z} & \frac{\tau_{z\delta}}{I_z} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \frac{f_{\delta 4}}{m} & \frac{f_{\delta 4}}{m} & \frac{f_{\delta 4}}{m} & \frac{f_{\delta 4}}{m} \end{pmatrix} \begin{pmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \\ \delta_4 \end{pmatrix} \quad (61)$$

Furthermore, it is more convenient to control

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the quadrotor in terms of its maneuvers, not individual rotors. So, let us define δ_r , δ_p , δ_y , and δ_H as maneuvering throttle for roll, pitch, yaw, and heave maneuver. The relation is as follows:

$$\begin{pmatrix} \delta_r \\ \delta_p \\ \delta_y \\ \delta_H \end{pmatrix} = \begin{bmatrix} -1 & -1 & 1 & 1 \\ 1 & -1 & -1 & 1 \\ -1 & 1 & -1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{pmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \\ \delta_4 \end{pmatrix} \quad (62)$$

RESULTS AND DISCUSSIONS

A simulation model was built on the Simulink program. The simulation model was built based on nonlinear dynamics proposed in **Equation 37-48** and on linear dynamics proposed in **Equation 61** in form of state-space matrices. The model was given impulse input on roll, pitch, yaw, and elevation, each. The comparisons of the simulation results are presented as the following.

Figure 4 depicts the result of roll impulse input to the dynamics of quadrotor. By giving an impulse input at $t = 5$ s, quadrotor's roll angle (ϕ) were affected both for linear and nonlinear model. In fact, the resulting response for roll, pitch, and yaw angle were extremely similar, or even arguably no difference. The results of quadrotor's response subjected to a pitch impulse input is represented in **Figure 5**. The response of pitch angle for both linear and nonlinear model shows no difference, even the roll and yaw response show are not difference as well. However, compared to roll response due to roll impulse input, the response of pitch angle due to pitch impulse input is slower. This could happen as a result of quadrotor's geometry, where the value of I_{xx} is lower than I_{yy} , indicating that the span of the quadrotor is narrower with respect to x-axis rotation. More interestingly, as shown in **Figure 6**, the response of yaw angle due to yaw step input is apparently linear and keeps going. This is perhaps due to the nature of yaw maneuver that relies on action-reaction effect of the rotors. Regardless, similar with two previous results, all of the attitude response (roll angle, pitch angle, and yaw angle) show no difference for linear and nonlinear model.

Based on those findings, it was found that, when a quadrotor is subjected to a single attitude input, the linear and nonlinear model will produce similar result in terms of quadrotor's attitude response. However, if we see the elevation response, linear model and nonlinear model results are difference. The linear model tends to be constant still, while the nonlinear model captures the deviation of elevation due to each impulse input. Moreover, as can be seen in **Figure 7**, the quadrotor's elevation response due to elevation impulse input are also difference for linear and nonlinear model. The nonlinear model apparently gives slower response that the linear model since the variables in the corresponding equation are more in nonlinear model. Nonetheless, the

attitude response (roll, pitch, and yaw) is still extremely similar for both models.

To get deeper, **Figure 8** displays the response of quadrotor when being given a combination of impulse input on roll, pitch, and yaw simultaneously. Linear model and nonlinear model shows good agreements in term of roll and pitch response in which the results are almost the same. A difference slightly appears in yaw response; the response of linear model is slightly higher than the nonlinear model. Lastly, the linear model, once again, did not capture any elevation change, whereas the nonlinear model did. This indicates that, to obtain any elevation change or deviation, a nonlinear model should be preferred.

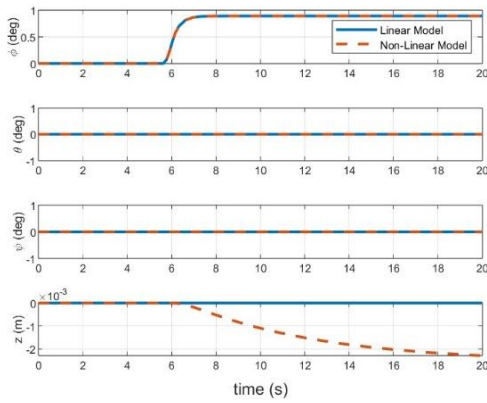


Figure 4 Response of roll step input

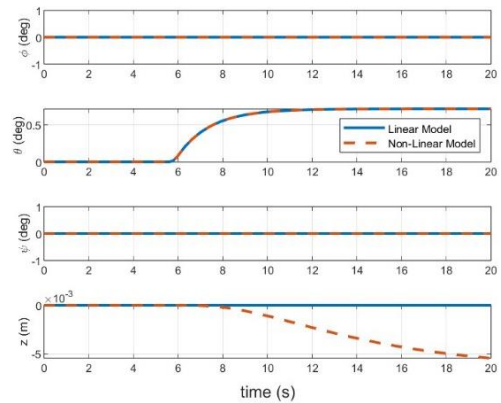


Figure 5 Response of pitch step input

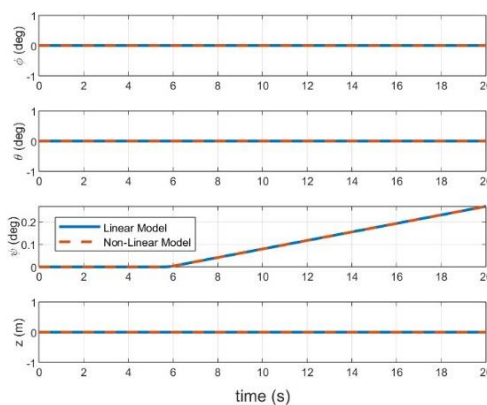


Figure 6 Response of yaw step input

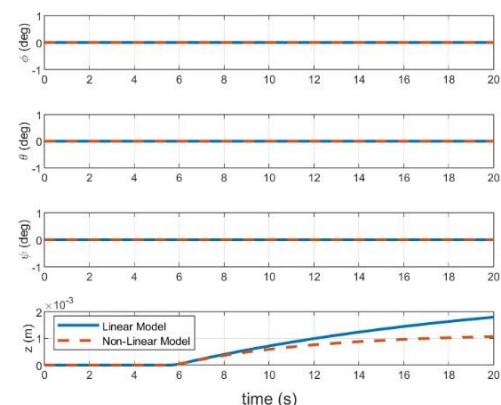


Figure 7 Response of elevation step input

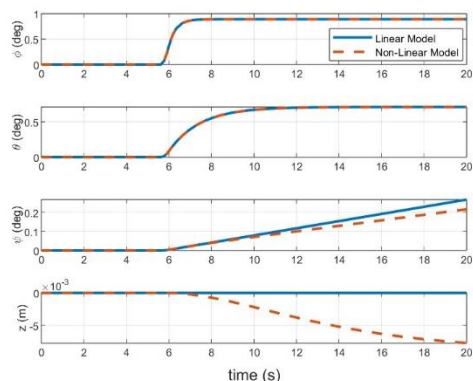


Figure 8. Response of attitudes (roll, pitch, and yaw) input

CONSLUSIONS AND SUGGESTIONS

In conclusion, the results obtained from linear and nonlinear simulation were similar. In fact, in most cases, the attitude response (roll, pitch, and yaw) for both simulations were nearly the same. The most notable difference was for the elevation response in which the linear simulation tended to be unable to capture the deviation, while the nonlinear simulation was able to. From these findings, it can be concluded that linear simulation is enough for analyzing quadrotor's attitude. However, if one is about to evaluate the elevation response of a quadrotor, then a nonlinear simulation would be better since it can track the deviation quite well.

However, this simulation was still not equipped with control algorithm. In reality, a quadrotor should have a control system to ensure that it is stable and controllable.

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